

Name:

Date:

Period: 6

Area of a Triangle (using trig) and Law of Cosines

Lesson Objective | WBAT find area of Δ s using trig ratios;
solve non-right Δ s using Law of Cosines.

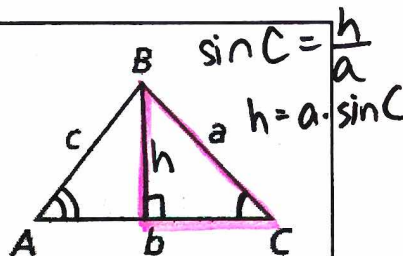
Area of a Triangle (Using Trig Ratios)

★SAS★

The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their included angle.

$$\text{Area} = \frac{1}{2}bh \rightarrow \text{area} = \frac{1}{2}b \cdot a \cdot \sin C$$

(or $\text{area} = \frac{1}{2}b \cdot c \cdot \sin A$)

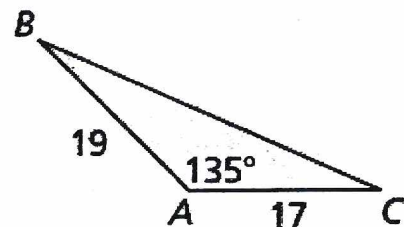


Example:

1. Find the area of the triangle. Round your answer to two decimal places.

$$\text{area} = \frac{1}{2}(19)(17)\sin 135$$

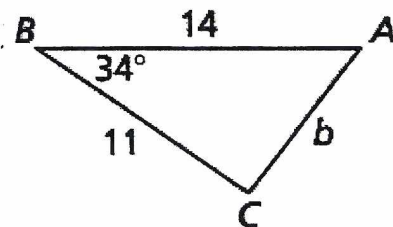
$$\approx 114.20$$



2. Find the area of the triangle. Round your answer to two decimal places

$$\text{area} = \frac{1}{2}(11)(14)\sin 34$$

$$\approx 43.06$$



Law of Cosines

You can use the Law of Cosines to solve triangles when...

- Two sides and the included angle are known (SAS case)
- All three sides are known (SSS case)

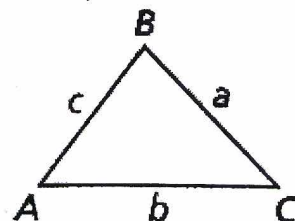
Law of Cosines (Theorem) (YouTube video of proof of this Theorem is on the classroom website)

The Law of Cosines can be written in either of the following forms for ΔABC with side lengths a (opposite $\angle A$), b (opposite $\angle B$) and c (opposite $\angle C$)

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$



Example:

SAS case

1. Solve the triangle. Round answers to two decimal places.

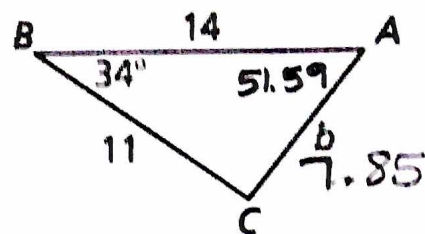
*must first solve for last side

$$b^2 = [11^2 + 14^2] - [2(11)(14)(\cos 34)]$$

$$b^2 = 317 - 255.3436$$

$$\sqrt{b^2} = \sqrt{61.6564}$$

$$\boxed{b \approx 7.85}$$



$$m\angle C = 180 - 34 - 51.59$$

$$\boxed{m\angle C \approx 94.41^\circ}$$

*must second solve for either the small or medium L.

$$\frac{\sin A}{11} = \frac{\sin 34}{7.85}$$

$$7.85 \sin A = 6.1511$$

$$\sin A = 0.7836$$

$$\rightarrow m\angle A = \sin^{-1}(0.7836)$$

$$\boxed{m\angle A \approx 51.59^\circ}$$

2. Solve the triangle. Round answers to two decimal places.

SSS case

*must solve first the biggest L

$$27^2 = [12^2 + 20^2] - [2(12)(20)(\cos B)]$$

$$729 = 544 - 480(\cos B)$$

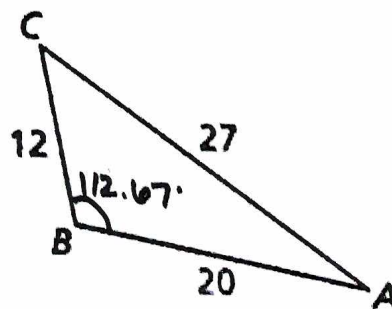
$$\cancel{729} = \cancel{544} - 480 \cos B$$

$$185 = -480(\cos B)$$

$$\cos B = -0.3854$$

$$m\angle B = \cos^{-1}(-0.3854)$$

$$\boxed{m\angle B \approx 112.67^\circ}$$



$$\frac{\sin A}{12} = \frac{\sin 112.67}{27}$$

$$\boxed{m\angle A \approx 24.21^\circ}$$

$$\boxed{m\angle C \approx 43.12^\circ}$$

by Δ sum Thm